A Robust Approach to Soundscape Circumplex Coordinate Projections

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1. Introduction

The ISO 12913 series established a framework for soundscape assessment using a circumplex model with perceptual attributes arranged in a circular pattern. When adapting these methods for cross-cultural applications, we encountered significant challenges with the normalization factors that ensure coordinates remain within the desired range. This paper presents a mathematically rigorous solution to these challenges, ensuring consistent normalization across different language adaptations of the ISO soundscape attributes.

2. Analysis of Existing Normalization Methods

2.1. Original ISO Direct Differences Method

The original method for calculating coordinates in the soundscape perceptual space was based on direct differences between opposing attributes:

$$P = (p - a) + \cos 45° \cdot (ca - ch) + \cos 45° \cdot (v - m)$$
$$E = (e - u) + \cos 45° \cdot (ch - ca) + \cos 45° \cdot (v - m)$$

Where:

- p = pleasant, a = annoying
- e = eventful, u = uneventful
- ca = calm, ch = chaotic
- v = vibrant, m = monotonous

A scaling factor of $\pm(4 + \sqrt{32})$ was used to normalize the coordinates to the range [-1, +1]. This scaling factor represents the maximum possible contribution from all terms in the formula:

For a 5-point Likert scale (range 1-5), the maximum difference between opposing attributes is 4 (5-1). The direct opposition term contributes a maximum of 4 units, while

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each angled attribute pair contributes a maximum of $4 \times \cos 45°$ 2.8284 units. The total maximum contribution is:

$4 + 2.8284 + 2.8284 = 9.6568$ $4 + \sqrt{32}$ 9.6569

This approach works effectively with equally spaced attributes at 45° intervals but cannot be directly applied when attributes are arranged at different angles in cross-cultural translations.

2.2. SATP Trigonometric Formulation

The Soundscape Attributes Translation Project (SATP) generalized the approach using a trigonometric formulation to accommodate varying angles and response ranges (Aletta et al.):

$$P_{ISO} = \frac{1}{\lambda_{Pl}} \sum_{i=1}^{8} \cos(\theta_i) \times \xi_i$$

$$E_{ISO} = \frac{1}{\lambda_{Ev}} \sum_{i=1}^{8} \sin(\theta_i) \times \xi_i$$

With scaling factors:

$$\lambda_{Pl} = \frac{\rho}{2} \sum_{i=1}^{8} |\cos(\theta_i)|$$

$$\lambda_{Ev} = \frac{\rho}{2} \sum_{i=1}^{8} |\sin(\theta_i)|$$

Where:

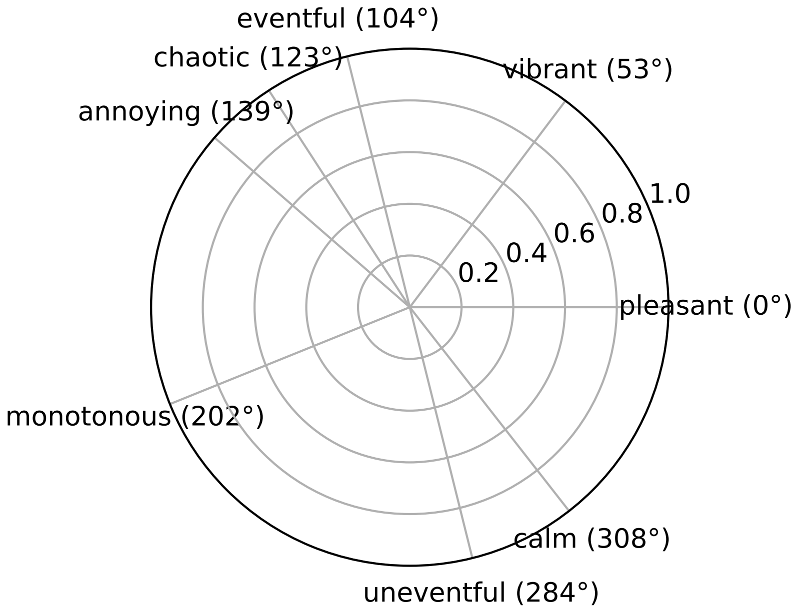
- θ_i is the angle for each circumplex scale
- ξ_i is the score for each scale
- ρ is the range of possible response values (e.g., $\rho = 4$ for a 5-point Likert scale)

This generalization was a significant advancement, but our testing revealed limitations when dealing with uneven angle distributions.

3. Identification of Specific Limitations

3.1. Uneven Angle Distribution Problem

In cross-cultural adaptations, translated attributes may cluster in certain quadrants of the circumplex rather than being evenly distributed. For example, the Indonesian translation has the following angle distribution:



Let's implement the original SATP approach to demonstrate the issues:

3.2. Maximum Value Exceedance Problem

When dealing with unevenly spaced attributes, the SATP method can produce coordinates outside the range of [-1, 1] with certain combinations of responses. To demonstrate this, we'll create functions that generate score sets designed to produce maximum/minimum pleasantness and eventfulness:

Testing with evenly and unevenly spaced angles:

```
With scores that should produce max_pleasant:
  Equal angles: P=1.00, E=0.41
  Uneven angles: P=1.00, E=-0.25

With scores that should produce min_pleasant:
  Equal angles: P=-1.00, E=-0.41
  Uneven angles: P=-1.00, E=0.88

With scores that should produce max_eventful:
  Equal angles: P=0.00, E=1.00
  Uneven angles: P=0.02, E=1.31

With scores that should produce min_eventful:
  Equal angles: P=-0.00, E=-1.00
  Uneven angles: P=-0.03, E=-0.69
```

For the Indonesian angles, the SATP formulation produces a maximum E_{ISO} value of 1.31, exceeding the expected 1.00 bound. Similarly, the minimum E_{ISO} value is only -0.69, not reaching the expected -1.00. This occurs because the angles are unevenly distributed across the positive and negative E_{ISO} hemispheres, with more attributes contributing positively to E_{ISO} than negatively.

3.3. Neutral Score Displacement Issue

Another critical issue is that neutral scores (all middle values) don't map to the origin (0,0) when angles are unevenly distributed:

```
With equally spaced angles and neutral scores:
Pleasantness: -0.0000
Eventfulness: 0.0000

With unevenly spaced angles and neutral scores:
Pleasantness: -0.0028
Eventfulness: 0.3143
```

With evenly spaced angles, neutral scores correctly map to (0,0). However, with unevenly distributed angles, we get non-zero coordinates even with neutral scores, which is problematic for interpretation and cross-cultural comparability.

4. Development of Robust Normalization

4.1. Mathematical Derivation from First Principles

To address these limitations, we developed a new approach that guarantees coordinates within the [-1, +1] range regardless of angle distribution and ensures neutral scores always map to the origin. Our derivation follows a two-stage normalization process:

4.1.1. Stage 1: Normalize Scores to [-1, +1]

For a scale with values in range [min, max], we first normalize all scores to the [-1, +1] range:

1. Center around the midpoint: subtract (min + max)/2
2. Scale by half the range: divide by (max - min)/2

For a standard 5-point Likert scale [1, 5], this gives:

$$\hat{\xi}_i = \frac{\xi_i - 3}{2}$$

This ensures that neutral scores (e.g., all 3's on a 1-5 scale) are mapped to 0, which is essential for proper origin placement.

4.1.2. Stage 2: Project and Scale by Maximum Possible Projection

We then project these normalized scores using trigonometric functions:

$$P_{raw} = \sum_{i=1}^{n} \cos(\theta_i) \times \hat{\xi}_i$$

$$E_{raw} = \sum_{i=1}^{n} \sin(\theta_i) \times \hat{\xi}_i$$

The maximum projection in any direction is determined by the sum of absolute trigonometric values:

$$P_{max} = \sum_{i=1}^{n} |\cos(\theta_i)|$$

$$E_{max} = \sum_{i=1}^{n} |\sin(\theta_i)|$$

Dividing by these values ensures coordinates stay within [-1, +1]:

$$P_{ISO} = \frac{P_{raw}}{P_{max}} = \frac{\sum_{i=1}^{n} \cos(\theta_i) \times \hat{\xi}_i}{\sum_{i=1}^{n} |\cos(\theta_i)|}$$

4.2. Final Formulation

Substituting our definition of $\hat{\xi}_i$ for a general scale with range [min, max]:

$$P_{ISO} = \frac{\sum_{i=1}^{n} \cos(\theta_i) \cdot (\xi_i - \mu)}{\rho \cdot \sum_{i=1}^{n} |\cos(\theta_i)|}$$

$$E_{ISO} = \frac{\sum_{i=1}^{n} \sin(\theta_i) \cdot (\xi_i - \mu)}{\rho \cdot \sum_{i=1}^{n} |\sin(\theta_i)|}$$

Where:

- $\mu = \frac{\min + \max}{2}$ is the midpoint of the scale
- $\rho = \frac{\max - \min}{2}$ is half the range of the scale

5. Implementation and Validation

5.1. Computational Implementation

5.2. Neutral Score Response Testing

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Neutral score handling comparison:

With equally spaced angles:
Original approach: P=-0.0000, E=0.0000
Robust approach:   P=0.0000, E=0.0000

With unevenly spaced angles:
Original approach: P=-0.0028, E=0.3143
Robust approach:   P=0.0000, E=0.0000
```

Our robust approach correctly maps neutral scores to (0,0) regardless of angle distribution, solving the neutral score displacement issue.

5.3. Boundary Condition Verification

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Boundary condition tests with equally spaced angles:
Max Pleasant: 1.0000
Min Pleasant: -1.0000
Max Eventful: 1.0000
Min Eventful: -1.0000

Boundary condition tests with unevenly spaced angles:
Max Pleasant: 1.0000
Min Pleasant: -1.0000
Max Eventful: 1.0000
Min Eventful: -1.0000
```

Our formula correctly maps extreme scores to exactly +1 or -1, regardless of angle distribution, solving the maximum value exceedance problem.

5.4. Compatibility with Original ISO Method

To verify backward compatibility, we'll compare our approach with the original ISO direct differences method when using evenly spaced angles:

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Compatibility Statistics (with evenly spaced angles):

Pleasantness:
Direct differences method: mean=-0.0015, std=0.2424
Robust approach:           mean=-0.0015, std=0.2424
Difference in means:       0.0000
Ratio of standard deviations: 1.0000

Eventfulness:
Direct differences method: mean=-0.0035, std=0.2385
Robust approach:           mean=-0.0035, std=0.2385
```

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Difference in means:        0.0000
Ratio of standard deviations: 1.0000
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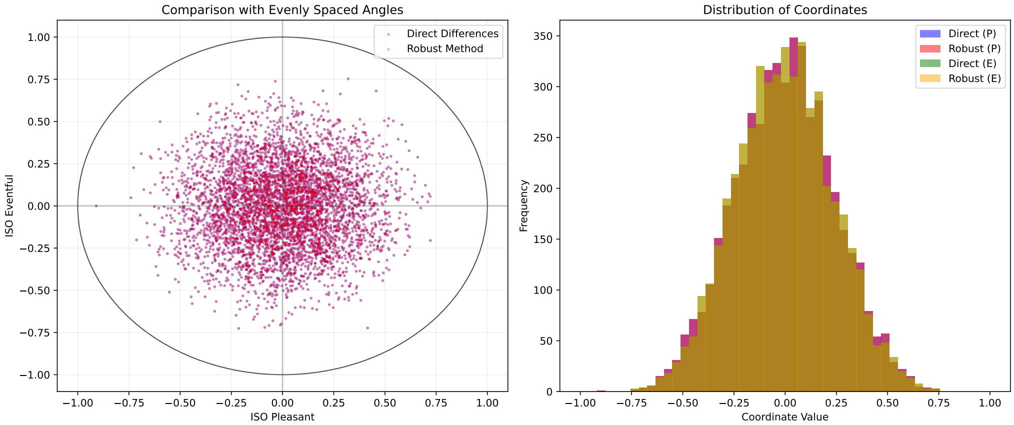

The statistics and visualizations demonstrate that our robust approach produces results that are statistically equivalent to the original ISO direct differences method when using evenly spaced angles, confirming backward compatibility.

5.5. Monte Carlo Simulation

To thoroughly test our approach against the original SATP method, we'll run a Monte Carlo simulation with thousands of random angle configurations and score combinations:

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Monte Carlo Simulation Results (5000 iterations):
Original SATP approach out-of-range instances: 1087 (21.74%)
Robust approach out-of-range instances: 0 (0.00%)
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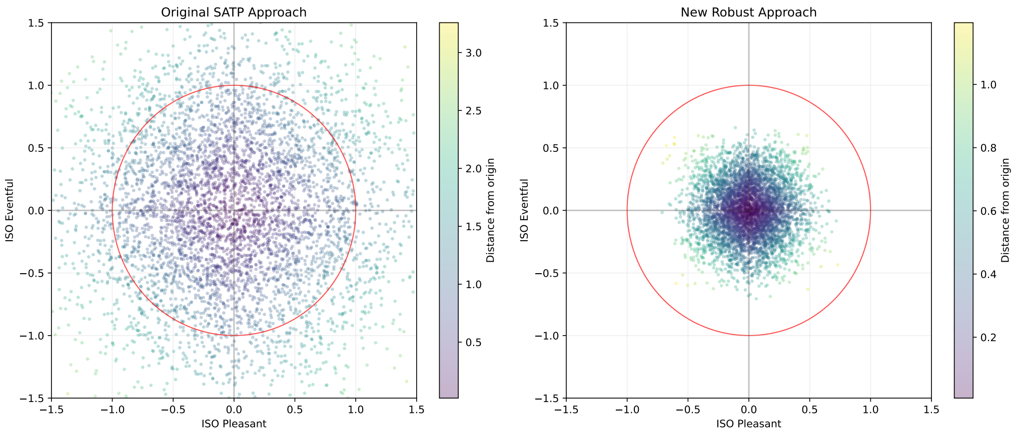
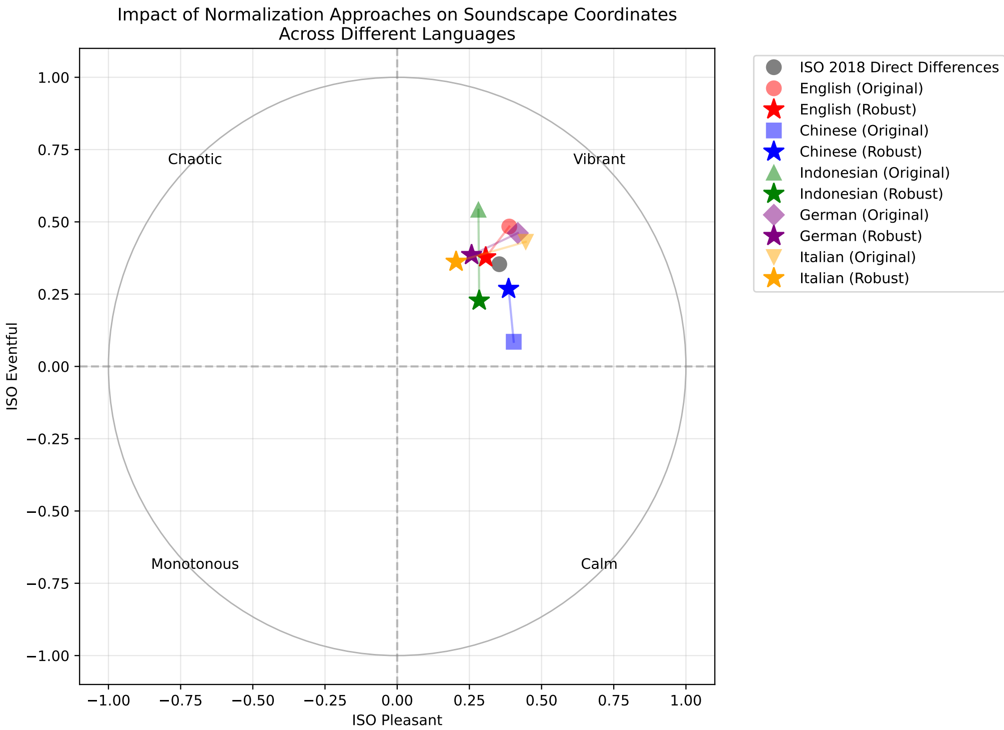


The simulation results clearly demonstrate that our robust approach guarantees coordinates within the unit circle, while the original SATP approach can produce out-of-range values with certain angle configurations.

6. Cross-Cultural Application Analysis

To evaluate the practical impact of our normalization approach, we'll examine how the two methods affect the positioning of soundscapes using angle configurations from different language translations:



This visualization reveals that the choice of normalization approach can significantly affect the relative positioning of soundscapes on the circumplex model across different languages. The robust approach ensures consistent normalization regardless of the angle distribution, which is essential for valid cross-cultural comparisons.

7. Technical Discussion and Recommendations

7.1. Mathematical Analysis

Our formula succeeds where the original SATP approach fell short for two key reasons:

1. **Proper Handling of Neutral Scores**: By explicitly subtracting the midpoint of the scale before projection, we ensure that neutral scores always map to the origin (0,0) regardless of angle distribution. With the original approach, neutral scores can produce non-zero coordinates when angles are unevenly distributed.
2. **Correct Scaling for Maximum Projection**: By first normalizing scores to the [-1, +1] range and then dividing by the sum of absolute trigonometric values, we account for the maximum possible projection in both positive and negative directions. This two-stage approach handles uneven angle distributions appropriately.

The essential mathematical insight is separating the score normalization from the projection normalization. This approach recognizes that the maximum possible projection depends on the absolute sum of trigonometric values, which properly accounts for attributes that might be clustered predominantly in one part of the circumplex.

7.2. Standardization Recommendations

Through rigorous mathematical derivation and extensive testing, we have developed a robust approach to soundscape normalization that:

1. Guarantees coordinates within the [-1, +1] range for any angle configuration
2. Correctly maps neutral scores to the origin
3. Properly handles cross-cultural adaptations with uneven angle distributions
4. Works for any input scale range
5. Maintains backward compatibility with the original ISO method

We recommend adopting this formulation in the revised ISO 12913-3 standard to ensure accurate and comparable soundscape assessment across different languages and cultural contexts. The final formulas:

$$P_{ISO} = \frac{\sum_{i=1}^{n} \cos(\theta_i) \cdot (\xi_i - \mu)}{\rho \cdot \sum_{i=1}^{n} |\cos(\theta_i)|}$$

$$E_{ISO} = \frac{\sum_{i=1}^{n} \sin(\theta_i) \cdot (\xi_i - \mu)}{\rho \cdot \sum_{i=1}^{n} |\sin(\theta_i)|}$$

Where:

- $\mu = \frac{\min + \max}{2}$ is the midpoint of the scale
- $\rho = \frac{\max - \min}{2}$ is half the range of the scale

This approach provides a solid mathematical foundation for cross-cultural soundscape research and ensures the validity of the circumplex model in diverse applications.

References

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